Reg. No: | | | | | | | | | |

G.T.N. ARTS COLLEGE (AUTONOMOUS)

(Affiliated to Madurai Kamaraj University)

(Accredited by NAAC with 'B' Grade)

END SEMESTER EXAMINATION - NOVEMBER 2019

Programme : M. Sc. Mathematics **Date: 13.11.2019**

Course Code: 17PCSC11 **Time: 10.00 a.m. to 1.00 p.m.**

Course Title : Mathematical Foundations **Max. Marks :75**

SECTION – A **[10 X 1 = 10]**

Answer ALL the Questions.

Choose the Correct Answer.

1. The dual of the statement $P \uparrow (Q \wedge \sim (R \downarrow P) \wedge T)$ is

   [a] $P \uparrow (Q \wedge (R \downarrow P) \vee T)$ [b] $P \uparrow (Q \wedge \sim (R \uparrow P) \wedge F)$

   [c] $P \downarrow (Q \wedge (R \downarrow P) \wedge T)$ [d] $P \downarrow (Q \vee \sim (R \uparrow P) \vee F)$

2. Pick out the well formed formula from the following

   [a] $\big(P \rightarrow (P \vee Q)\big)$ [b] $(P \vee Q) \wedge R \rightarrow (P \wedge Q) \vee (P \wedge R)$

   [c] $(P \wedge Q) \leftrightarrow P)$ [d] $P \rightarrow (Q \wedge R) \vee S$

3. A digraph in which every point has out degree one is called-----------------

   [a] Complete [b] Functional

   [c] Converse [d] Sub digraph

4. The number of lines in a complete graph G with 'n' points is

   [a] $\frac{n(n-1)}{2}$ [b] $n(n-1)$

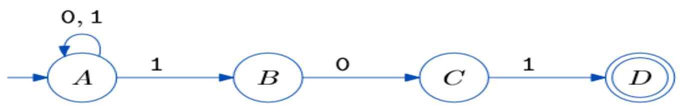
   [c] n [d] n^2

5. For a grammar G with productions $S \to SS, S \to aSb, S \to bSa, S \to \lambda$, which of the following holds true.

[a] $S \Rightarrow abba$ [b] $S \overset{*}{\Rightarrow} abba$

[c] $abba \notin L(G)$ [d] $S \overset{*}{\Rightarrow} aaa$

6. Pick out the string that are accepted by the following NFA



[a] 1101 [b] 0100

[c] 1111 [d] 0110

7. In a group $(R^*,*)$ define $a*b=\frac{ab}{2}$, then the identity element is

[a] 1 [b] 0

[c] 2 [d] $\frac{1}{2}$

8. The incorrect statement is

[a] Any cyclic group is abelian.

[b] Any abelian group is cyclic.

[c] The rule $(ab)^2 = a^2b^2$ is true in any abelian group.

[d] s_3 is a cyclic group.

9. If $p(x_1, x_2, x_3) = \bigoplus 3,5,6$ is a Boolean polynomial then its

[a] $p'(x_1, x_2, x_3) = \bigoplus 0, 1, 2, 4, 7,8$

[b] $p'(x_1, x_2, x_3) = \bigoplus\ 1, 2, 4, 7,8$

[c] $p'(x_1, x_2, x_3) = \bigoplus 0, 1, 2, 4, 7$

[d] $p'(x_1, x_2, x_3) = \bigoplus\ 1, 2, 4, 7$

10. Pick out the incorrect statement from the following:

[a] $(N, \leq)$ is a bounded lattice.

[b] $(P(X), \subseteq)$ is a lattice.

[c] N_5 is not a Modular lattice.

[d] Every Boolean algebra has at least two elements.

SECTION – B **[5 X 7 = 35]**

Answer ALL the Questions.

11.a) (i) Construct the truth table for $\sim (\sim P \wedge \sim Q)$.

(ii)Verify whether $(P \vee Q) \rightarrow P$ is a tautology or not.

[OR]

b) Show that $S \vee R$ is tautologically implied by

$(P \vee Q) \wedge (P \rightarrow R) \wedge (Q \rightarrow S)$.

12. a) (i) Prove that for a graph G, $\delta(G) \leq \deg(G) \leq \Delta(G)$ for all $v \in V(G)$.

(ii) Define the adjacency matrix with an example.

[OR]

b) Prove that a graph is a tree if and only if it is minimally connected.

13. a) Let $M = (\{q_0, q_1, q_2, q_3\}, \{a, b\}, \delta, q_0, \{q_1\})$ where δ is given by

$\delta(q_0, a) = q_1, \delta(q_0, b) = q_2$

$\delta(q_1, a) = q_3, \delta(q_1, b) = q_0$

$\delta(q_2, a) = q_2, \delta(q_2, b) = q_2$

$\delta(q_3, a) = q_2, \delta(q_3, b) = q_2$

i). Represent M by its state table.

ii). Represent M by its state diagram

iii). Which of the following strings are accepted by M? (a) ababa (b) aabba and (c) aaaab

[OR]

b) Construct a DFA for the language $L = \{w \,/\, |w| \text{ is even and } w \epsilon \{0,1\}^* \}$

14. a) A nonempty subset H of a group {G,*} will be a subgroup of G if and only if $a * b^{-1} \in H$, whenever $a, b \in H$.

[OR]

b) If a and b are the elements of a group {G,*}, then prove that $(a * b)^{-1} = b^{-1} * a^{-1}$.

15. a) Find the principal disjunctive normal form of

$p(x_1, x_2, x_3) = (x_2 + x_1 x_3)\overline{\left((x_1 + x_3)x_2\right)}$.

[OR]

b) (i) Prove that in any lattice $(L, \leq)$, the operations $\wedge$ and $\vee$ are isotone.

(ii) Prove that the lattice N_5 is not a Modular lattice.

SECTION – C **[3 X 10 = 30]**

Answer Any THREE Questions.

16. Show that $R \to S$ can be derived from the premises $P \to (Q \to S), \sim R \vee P$ and Q.

17. Prove that a simple graph with 'n' vertices and 'k' components can have at most $\frac{(n-k)(n-k+1)}{2}$ edges.

18. Construct an NFA accepting all strings ending with either 1010 or 001. Use it to construct a deterministic finite automaton accepting the given set.

19. Let H be a subgroup of a group G. Then prove that the following are equivalent.

(i) $Ha = aH$ for every $a \in G$.

(ii) $a^{-1}Ha = H$ for every $a \in G$.

$(iii) a^{-1}Ha \subset H$ for every $a \in G$.

20. Prove that $(L \times M, \wedge, \vee)$ is a lattice.

SECTION – C [3 X 10 = 30]

Answer Any THREE Questions.

16. State and prove Cauchy theorem.
17. Prove that two abelian groups of order p^n are isomorphic if and only if they have the same invariants.
18. If R is a commutative ring with unit element and M is an ideal of R, then prove that M is a maximal ideal of R if and only if R/M is a field.
19. State and prove Gauss Lemma.
20. Let $f(x) \in F[x]$ be of degree $n \geq 1$. Prove that there is an extension E of F of degree at most $n!$ in which $f(x)$ has n roots.

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END SEMESTER EXAMINATION - NOVEMBER 2019

Programme : M. Sc. Mathematics **Date: 13.11.2019**
Course Code:17PMAC11 **Time: 10.00 a.m. to 1.00 p.m.**
Course Title : Algebra – I **Max Marks :75**

SECTION – A [10 X 1 = 10]

Answer ALL the Questions.

Choose the Correct Answer.

1. Conjugacy is an __________ relation on G.

   [a]Reflexive [b] Equivalence
   [c] Symmetric [d] Transitive

2. If A, B are finite subgroups of G then $o(AxB) =$ ______

   [a] $\frac{o(A)o(B)}{o(A \cap xBx^{-1})}$ [b] $\frac{o(A)}{o(A \cap xBx^{-1})}$
   [c] $\frac{o(A)}{o(A \cap xBx^{-1})}$ [d] $\frac{o(A)o(B)}{o(xAx^{-1})o(B)}$

3. If A and B are groups, then $A \times B$ is isomorphic to _______

   [a] A [b] B X A
   [c] B [d] $A \cap B$

4. If G is an abelian group and S is any integer, then G(S)=______

   [a] $\{x \in G / x^s = e\}$ [b] $\{x \in G / x^e = e\}$
   [c] $\{x \in G / x = e\}$ [d] $\{x \in G / x^{-1} = e\}$

5. The only ideals of F are (is) _______

[a] (o) [b] F

[c] (o) or (F) [d] (o) and F

6. An Euclidian ring possesses a __________ element.

[a] inverse [b] unit

[c] commutative [d] ideal

7. If P is a prime number of the form 4n+1, then the congruence $x^2 \equiv$______

[a] 1 mod p [b] -1 mod p

[c] p mod 1 [d] -1 mod -p

8. If *f(x)* and *g(x)*are primitive polynomials. Then _______ is a primitive polynomial.

[a] *f(x)g(x)* [b] *f(x) o g(x)*

[c] *f(x)/g(x)* [d] *f(x)+g(x)*

9. If $a \in K$ is algebraic of degree n over F, then $[F(a):F] =$_________

[a] n^a [b] a^n

[c] n [d] a

10. τ^* defines an isomorphism of $F[x]$ onto $F'[t]$ with the property that ______ for every $\alpha \in F$.

[a] $\alpha\tau^* = \alpha'$ [b] $\alpha\tau^* = \alpha$

[c] $\alpha\tau^* = \tau^*$ [d] $\alpha\tau^* = \alpha\tau$

SECTION – B **[5 X 7 = 35]**

Answer ALL the Questions.

11. a) Prove that N(a) is a subgroup of G.

[OR]

b) Prove that $n(k) = 1 + p + ... + p^{k-1}$.

12. a) Suppose that G is the internal direct product of $N_1, ...N_n$. Then prove that for $i \neq j$, $N_i \cap N_j = (e)$ and if $a \in N_i$, $b \in N_j$ then $ab = ba$.

[OR]

b) Prove that the number of nonisomorphic abelian groups of order p^n, p a prime equals the number of partitions of n.

13. a) Let R be a communicative ring with unit element whose only ideals are (o) and R itself, then prove that R is a field.

[OR]

b) Prove that every integral domain can be imbedded in a filed.

14. a) If $f(x), g(x)$ are two nonzero elements of $F[x]$, then show that $\deg f(x)g(x) = \deg f(x) + \deg g(x)$

[OR]

b) Prove that $R[x]$ is an integral domain, if R is an integral domain.

15. a) If a and b in K are algebraic over F of degrees m and n, respectively, then show that $a \pm b, ab$ and a/b *if* $(b \neq 0)$ are algebraic over F of degree at most mn.

[OR]

b) If $p(x) \in F[x]$ and if K is an extension of F, then for any element $b \in k$, show that $p(x) = (x-b)q(x) + p(b)$ where $q(x) \in K[x]$ and where $\deg q(x) = \deg p(x) - 1$.

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END SEMESTER EXAMINATION - NOVEMBER 2019

Programme : M. Sc. Mathematics | **Date : 15.11.2019**
Course Code: 17PMAC12 | **Time: 10.00 a.m. to 1.00 p.m.**
Course Title : Analysis - I | **Max Marks :75**

SECTION – A [10 X 1 = 10]

Answer ALL the Questions.

Choose the Correct Answer.

1. A sequences $\{s_n\}$ of real number is said to be monotonically increasing if ________

   [a] $s_n \le s_{n+1}$ [b] $s_n \ge s_{n+1}$

   [c] $s_n < s_{n+1}$ [d] $s_n > s_{n+1}$

2. $\lim_{n \to \infty} \left(1+\frac{1}{n}\right)^n =$ ____________

   [a] e^n [b] e

   [c] e^{-1} [d] e^{-n}

3. The product of two converges series is__________

   [a]converges [b]diverges

   [c]converges and diverges [d] increasing sequences

4. If $\sum a_n$ is a series of complex numbers which converges absolutely then every rearrangement of $\sum a_n$ __________

[a] Diverges [b]Converges

[c] Continuous [d] bounded

5. Every uniformly continuous is__________

[a] Converges and Continuous [b] Continuous

[c]not continuous [d] not converges

6. A mapping f of a set E into R^k is said to be bounded if there is a real number M such that _______

[a] $|f(x)| \geq M$ [b] $|f(x)| \leq M$

[c] $|f(x)| \neq M$ [d] $|f(x)| = M$

7. Monotonic functions have no ________of second kind

[a] Continuous [b]Uniformly continuous

[c] Discontinuities [d] converges

8. The function $f(x) = \begin{cases} 1 & if \quad x \;\; is \;\; rational \\ 0 & if \quad x \;\; is \;\; irrational \end{cases}$ then ___________

[a] f has a discontinuity of second kind at every point x

[b] f has a continuity of second kind at every point x

[c] f has a continuous at x = 0

[d] f has a continuous at x = 0

9. Let f be defined on [a,b], if f has a local maximum at a point $x \in (a,b)$ and if $f'(x)$ exists then ________

[a] $f'(x) \neq 0$ [b] $f'(x) = 0$

[c] $f'(x) > 0$ [d] $f'(x) < 0$

10. Let f be defined on [a b].If f is differentiable at a point $x \in [a,b]$ then f is __________at x

[a] continuous [b]uniformly continuous

[c]bounded [d] converges

SECTION – B **[5 X 7 = 35]**

Answer ALL the Questions.

11. a) Let $\{P_n\}$ be a sequences in a metric space X.

(i) If $p \in X$, $p' \in X$ and if $\{P_n\}$converges to p and to p' then prove that $p = p'$

(ii) If $\{P_n\}$ converges to p , then prove that $\{P_n\}$is bounded

[OR]

b) If $\{P_n\}$ is a sequence in a compact metric space X then prove that some subsequence of $\{P_n\}$ converges to a point of X.

12. a) Suppose (i) the partial sums A_n of $\sum a_n$ form a bounded sequence

(ii) $b_0 \geq b_1 \geq b_2 \geq \ldots\ldots$

(iii) $\lim_{n \to \infty} b_n = 0$

Then prove that $\sum a_n b_n$ converges

[OR]

b) Define Rearrangements with example

13. a) Prove that a mapping f of a metric space X into a metric space Y is continuous on X iff $f^{-1}(V)$ is open in X for every open set V in Y.

[OR]

b) If f is a continuous mapping of a compact metric space X into a metric space Y then prove that $f(X)$ is compact.

14. a) If f is a continuous mapping of a compact metric space X into a metric space Yand E isa connected subset of X then prove that $f(E)$ is connected.

[OR]

b) State and prove intermediate value Theorem.

15. a) State and prove Mean Value Theorem.

[OR]

b) Suppose f is a real differentiable function on [a, b]and suppose $f'(a) < \lambda < f'(b)$. Then prove that there is a point $x \in (a,b)$ such that $f'(x) = \lambda$.

SECTION – C [3 X 10 = 30]

Answer Any THREE Questions.

16. Prove that the following:

a) In a metric space x ,every convergent sequence is a Cauchy sequence.

b) If X is a compact metric space and if $\{P_n\}$ is a Cauchy sequence in X then $\{P_n\}$ converges to some point of X.

c) In R^k ,every Cauchy sequence converges.

17. Suppose (i) $\sum_{n=0}^{\infty} a_n$ converges absolutely (ii) $\sum_{n=0}^{\infty} a_n = A$ (iii) $\sum_{n=0}^{\infty} b_n = B$

(iv) $\sum_{k=0}^{n} a_k b_{n-k}$ (n=0,1,2,3,….) Then prove that $\sum_{n=0}^{\infty} c_n = AB$

18. Let f be continuous mapping of a compact metric space X into a metric space Y. Then prove that f is uniformly continuous on X

19. Let f be monotonic on (a,b).Then prove that the set of points of (a,b) at which f is discontinuous is at most countable.

20. State and prove Taylor's Theorem.

SECTION – C [3 X 10 = 30]

Answer Any THREE Questions.

16. State and prove Abel's formula.

17. Solve $(X^2 D^2 – XD +2) = x \log x$.

18. Solve $(X^2D^2 +3X D +1)y =1/(1- X)^2$.

19. Use Picard's method to obtain a solution of the differential equation $y'=x^2 –y, y(0)=0$. Find a least 4^{th} approximation to each solution.

20. Find the eigen values and eigen functions of the shrum –Lioville problem $X''(x) +\lambda X=0, X'(0)=0, X'(L)=0$.

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END SEMESTER EXAMINATION - NOVEMBER 2019

Programme :M. Sc., Mathematics **Date: 18.11.2019**

Course Code:17PMAC13 **Time: 10.00a.m. to 1.00p.m.**

Course Title : Ordinary Differential Equations **Max Marks :75**

SECTION – A [10 X 1 = 10]

Answer ALL the Questions.

Choose the Correct Answer.

1. If $y_1 (t) =\sin t$ and $y_2 (t) =1-t$ are solutions of a second order differential equations then $W(y_1,y_2)$ is ________.

   [a] (t-1)cost+sin t [b] (t-1) cost- sin t

   [c] (t-1)cos t + sin t [d] (t-1) cos t – sin t.

2. If the Wronskian W of two function φ_1,φ_2 vanishes at some $x_0\in I$, then in the whole interval I, _______.

   [a] W=0 [b] $W\neq0$ except at x_0

   [c] W=1 [d] W>0.

3. The homogeneous linear equations is also known as ________.

   [a] linearly dependent [b] linearly independent

   [c] Cauchy euler equation [d] linear combination.

4. The complementary function of $x^2y''+ 5xy' +4y = x \log x$.

   [a] $(c_1+c_2)x^{-2}$ [b] $(c_1+c_2 \log x)/x^2$

   [c] $x^2 (c_1 +c_2 \log x)$ [d] $(c_1 + c_2) x^2$.

5. The Particular integral of $(D^3-D^2-D+1)y = e^{-2z}$ is _______.

[a] $1/9e^{-2z}$ [b] $-1/9\, e^{-2z}$

[c] $9e^{2z}$ [d] $1/9\, e^{-29z}$

6. If $f(-a^2)=0$ then $1/(D^2+a^2) \sin ax$ =_______

[a] $x/2a \cos ax$ [b] $-2a\cos ax$

[c] $-x/2a \cos ax$ [d] $-x/4a \cos ax$

7. The two conditions of the second order initial value problem are_______.

[a] $y(x_0)=k$, $y'(x_0)= -1$ [b] $y(x_0)=x$, $y'(x_0)=l$

[c] $y(x_0)=k$, $y'(x_0)=l$ [d] $y(x)=k$, $y'(x)=l$

8. The n^{th} approximation y_n is ________.

[a] $y_n(x) = y_o + \int_{x_0}^{x} f(x, y_{n-1})\, dx$ [b] $y_n(x) = y_o + \int_{x_0}^{x} f(x, y_n)\, dx$

[c] $y_n(x) = y_o + \int_{x}^{x_0} f(x, y_{n-1})\, dx$ [d] $y_n(x) = y_o + \int_{x}^{x_0} f(x, y_n)\, dx$

9. The eigen functions corresponding to differential eigen values are orthogonal with respect to some______function.

[a] weight [b] odd

[c] even [d] unweight

10. $|f(x,y_2)-f(x,y_1)| \le K|y_2 - y_1|$ is ____

[a] Lipschitz constant [b] Cauchy condition

[c] Cauchy constant [d] Lipschitz condition

SECTION – B **[5 X 7 = 35]**

Answer ALL the Questions.

11. a) Show that $y=3e^{2x} + e^{-2x} - 3x$ is the unique solution of the initial value problem $y''-4y=12x$ where $y(0)=4$ and $y'(0)=1$.

[**OR**]

b) If $y_1(x) = \sin 3x$ and $y_2(x) = \cos 3x$ are two solutions of $y''+ay=0$, then show that $y_1(x)$, $y_2(x)$ are linearly independent solutions.

12. a) Solve $x^2y_2 + xy_1 - 4y=0$.

[**OR**]

b) Find the values of λ for which all solutions of $x^2y'' - 3xy' - \lambda y = 0$ tend to zero; $x\to\infty$.

13. a) Solve $(x+1)^2y'' - 4(x+1)y' + 6y = 6(x+1)$.

[**OR**]

b) Solve $[(1+2x)^2D^2 - 6(1+2X)D+16]\, Y = 8(1+2X)^2$

14. a) Find the third approximation of the solution of the equation $y'' = x^2 y' + x^4 y$, where $y = 5$ and $y' = 1$ when $x=0$.

[**OR**]

b) Find the third approximation of the solution of the equations $y' = z$, $z' = x^2 z + x^4 y$ by picard method $y= 5$ and $z=1$ when $x=0$.

15.a) Define Lipschitz condition and Lipschitz Constants.

[**OR**]

b) Show that the solution of the initial value problem $dy/dx = f(x,y)$, $y(x_0) = y_o$ may not be unique although $f(x,y)$ is continuous.

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END SEMESTER EXAMINATION - NOVEMBER 2019

Programme : M. Sc. Mathematics **Date: 20.11.2019**
Course Code: 17PMAC14 **Time: 10.00 a.m. to 1.00 p.m.**
Course Title :Numerical Analysis **Max Marks :75**

SECTION – A **[10 X 1 = 10]**

Answer ALL the Questions.

Choose the Correct Answer.

1. In the Gauss elimination method for solving a system of linear algebraic equations ______.

   [a] Diagonal matrix [b] Lower diagonal matrix

   [c] Upper diagonal matrix [d] Singular matrix

2. Jacobi's method is also known as _________.

   [a] Displacement method [b] Simultaneous displacement method

   [c] Simultaneous method [d] Diagonal matrix

3. Using Bisection method negative root of $x^3 - 4x + 9 = 0$ correct to three decimal plane is ________.

   [a] 2.506 [b] 2.706

   [c] 2.406 [d] 2.606

4. Errors may occur in performing numerical computation on the computer due to__________.

   [a] Rounding errors [b] power fluctuation
   [c] operator fatigue [d] not updation

5. In general, the ratio of truncation error to that of round off error is _____.

   [a] 2:1 [b] 1:1
   [c] 1:2 [d] 1:3

6. The convergence of which of the following method is sensitive to starting value?

   [a] False position [b] Gauss seidal method
   [c] Newton-Raphson method [d] Bisection method

7. Interpolation provides a mean for estimating function

   [a] At the beginning points [b] At the ending point
   [c] At the intermediate point [d] At the exact point

8. Gaussion process is a _________ interpolation process.

   [a] Linear [b] nonlinear
   [c] notan interpolation [d] bilinear

9. The order of errors the Simpson's rule for Numerical integration with a step size h is

   [a] h [b] h^2
   [c] h^3 [d] h^4

10. The accuracy of trapezoidal rule is _________.

   [a] least accurate [b] highly varied
   [c] exact [d] most accurate

SECTION – B [5 X 7 = 35]

Answer ALL the Questions.

11. a) Perform five iterations of the bisection method to obtain a root of the equation $f(x) = \cos x - xe^x = 0$.

[OR]

b) Perform three iterations of the multipoint iteration method, to find the root of the equation $f(x) = \cos x - xe^x = 0$

12. a) Solve the equations $x_1 + x_2 + x_3 = 6,\ 3x_1 + 3x_2 + 4x_3 = 20,$ $2x_1 + x_2 + 3x_3 = 13$ using the Gauss elimination method.

[OR]

b) Prove that no eigen value of a matrix A exceeds the norm of a matrix.

13. a) Using the following values of $f(x)$ and $f'(x)$

x	$f(x)$	$f'(x)$
-1	1	-5
0	1	1
1	3	7

Estimate the values of $f(-0.5)$ and $f(0.5)$ using piecewise cubic Hermite interpolation.

[OR]

b) Find the unique polynomial of degree 2 or less, such that $f(0) = 1, f(1) = 3, f(3) = 55$ using the iterated interpolation.

14. a) The following table of values is given

$x:$	-1	1	2	3	4	5	7
$f(x):$	1	1	16	81	256	625	2401

Using the formula $f'(x_1) = (f(x_2) - f(x_0))/2h$ and the Richardson extrapolation.

[OR]

b) Find the approximate value of $I = \int_0^1 \frac{\sin x}{x} dx$ using (i) mid-point rule

(ii) two-point open type rule.

15. a) Convert the following second order initial value problem into a system of first order initial value problem $ty'' - y' + 4t^3 y = 0,\ y(1) = 1, y'(1) = 2$

[OR]

b) Find the general solution of the difference equations

$\Delta^2 u_n + \Delta u_n + (1/4)u_n = 0$. Is the solution bounded?

SECTION – C **[3 X 10 = 30]**

Answer Any THREE Questions.

16. The equation $f(x) = 3x^3 + 4x^2 + 4x + 1 = 0$ has a root in the interval $(-1, 0)$ Determine an iteration function $\varphi(x)$, such that the sequence of iteration obtained from $x_{k+1} = \varphi(x_k)$, $x_0 = -0.5, K = 0,1.....$ converges to the root.

17. Show that the matrix $\begin{bmatrix} 12 & 4 & -1 \\ 4 & 7 & 1 \\ -1 & 1 & 6 \end{bmatrix}$ is positive definite.

18. Obtain the piecewise quadratic interpolation polynomial for the function $f(x)$ defined by the data.

$x:$	-3	-2	-1	1	3	6	7
$f(x):$	369	222	171	165	207	990	1779

Hence, find an approximation value of $f(-2.5)$ and $f(6.5)$.

19. Evaluate the integral $I = \int_1^2 \int_1^2 \frac{dxdy}{x+y}$ using the trapezoidal rule with $h = k = 0.5$ and $h = k = 0.25$. Improve the estimate using Romberg integration.

20. Solve the initial value problem $u' = -2tu^2, u(0) = 1$ with $h = 0.2$ on the interval $[0, 0.4]$. Use the fourth order classical Runge-Kutta method. Compare with the exact solution.

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END SEMESTER EXAMINATION - NOV 2019

Programme: M. Sc., Mathematics **Date: 13.11.2019**

Course Code: 17PMAC21 **Time: 2.00 p.m. to 5.00 p.m.**

Course Title : Algebra – II **Max. Marks :75**

SECTION – A **[10 X 1 = 10]**

Answer ALL the Questions.

Choose the Correct Answer.

1. If f is of characteristic o and if a,b are algebraic over F, then there exit $C \in F(a,b)$ such that $F(a,b) =$ _______

   [a] $F(c)$ [b] $F(b,a)$

   [c] $F(a,c)$ [d] $F(b,c)$

2. The fixed field of G is a _____of K.

   [a] Finite Filed [b] Separate Field

   [c] Filed [d] Subfield

3. If T satisfies a polynomial $h(x)$ other than the nominal polynomial $p(x)$, then which one of the following is true?

   [a] $h(x) / p(x)$ [b] $p(x) / h(x)$

   [c] $p(x)\ h(x)$ [d] $h(x)\ \ p(x)$

4. If $T \in A(V)$, then $\lambda \in F$ is called _____ of T if $\lambda - T$ is singular.

[a] scalar [b] eigen value

[c] invertible [d] eigen vector

5. If A is triangular, then its characteristic roots are precisely the elements on the ______

[a] first row [b] first column

[c] main diagonal [d] upper diagonal

6. If V is an dimensional over F and if $T \in A(V)$ has all its ________in F, then T satisfies a polynomial of degree n over.

[a] characteristic roots [b] equal roots

[c] isomorphic [d] not equal

7. If $T \in A(V)$ is nilpotent and $T^k = 0$, but $T^{k-1} \neq 0$ then k is called ____

[a] index of nilpotent [b] index of $A(V)$

[c] index of linear transformation [d] index of T

8. If M of dimension m is cyclic with respect to T, then the dimension of MT^K is _____

[a] $K - m$ [b] $m - K$

[c] $\frac{m-K}{2}$ [d] $\frac{m+K}{2}$

9. $t_r(A+B)=$_____

[a] $t_r(A)+t_r(B)$ [b] $t_r(A)-t_r(B)$

[c] $t_r(A)t_r(B)$ [d] $t_r(A)/t_r(B)$

10. If $T \in A(V)$ is hermitian, the all its characteristics roots are ______

[a] complex [b] real

[c] real and complex [d] none of these

SECTION – B **[5 X 7 = 35]**

Answer ALL the Questions.

11. a) Prove that the polynomial $f(x) \in F[x]$ has a multiple root if and only if $f(x)$ and $f'(x)$ have a nontrivial common factor.

[OR]

b) If K is a finite extension of F, then prove that $G(K,F)$ is a finite group and its order, $o(G(K,F))$ satisfies $o(G(K,F)) \leq [K:F]$

12. a) Let A be an algebra, with unit element, over F, and suppose that A is of dimension m over F. Then prove that every element in A satisfies some nontrivial polynomial in $F[x]$ of degree atmost m.

[OR]

b) If $\lambda \in F$ is characteristic root of $T \in A(V)$, then prove that λ is a root of the minimal polynomial of T. In particular, prove that T only has a finite number of characteristic roots in F.

13. a) Let F be a field and let V be the set of all polynomials in x of degree $n-1$ or less over F on V. Let D defined by

$$(\beta_0 + \beta_1 x + ... + \beta_{n-1} x^{n-1})D = \beta_1 + 2\beta_2 x + ... + c\beta_i x^{i-1} + ...(n-1)\beta_{n-1} x^{n-2})D$$

Find the matrix of D.

[OR]

b) If V is dimensional over F and if $T \in A(V)$ has all its characteristic roots in F, then prove that T satisfies a polynomial of degree n over F.

14. a) Suppoe that $V = V_1 \oplus V_2$, where V_1 and V_2 are subspaces of V invariant under T. Let T_1 and T_2 be the linear transformations induced by T on V_1 and V_2 respectively. If the minimal Polynomial of T_1 over F is $P_1(x)$ and that T_2 over F is $P_2(x)$, then prove that the minimal Polynomial for T over F is the least common multiple of $P_1(x)$ and $P_2(x)$.

[OR]

b) Suppose the two matrices A, B in F_n are similar in K_n where K is an extension of F. Then prove that A and B are already similar in F_n.

15. a) For $A, B \in F_n$ and $\lambda \in F$, prove that

i) $t_r(\lambda A) = \lambda t_r(A)$ ii) $t_r(A+B) = t_r A + t_r B$ and iii) $t_r(AB) = t_r(BA)$

[OR]

b) Prove that the linear transformation T in V is unitary if and only if it takes an orthonormal basis of V into an orthonormal basis of V.

SECTION – C [3 X 10 = 30]

Answer Any THREE Questions.

16. If $P(x) \in F[x]$ is solvable by radicals over F, then prove that the Galois group over F of $p(x)$ is a solvable group.

17. If $\lambda_1, \lambda_2, ...\lambda_k$ in F are distinct characteristic roots of $T \in A(V)$ and if $v_1, v_2, ...v_k$ are characteristic vectors of T belonging to $\lambda_1, \lambda_2, ...\lambda_k$ respectively, then prove that $v_1, v_2, ...v_k$ are linearly independent over F.

18. If $T \in A(V)$ has all its characteristic roots in F, then prove that there is a basis of V in which the matrix of T is triangular.

19. Prove that there exists a subspace W of V, invariant under T, such that $V = V_1 \oplus W$.

20. Prove that A is invertible if and only if $\det A \neq 0$.

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END SEMESTER EXAMINATION - NOVEMBER 2019

Programme: M. Sc. Mathematics
Course Code: 17PMAC22
Course Title : Analysis - II

Date: 15.11.2019
Time: 2.00 p.m. to 5.00 p.m.
Max. Marks :75

SECTION – A **[10 X 1 = 10]**

Answer ALL the Questions.

Choose the Correct Answer.

1. The unit step function I is defined by $I(x)$ is o if ________.

   [a] $X = 0$ [b] $X < 0$

   [c] $X \leq 0$ [d] $X \neq 0$

2. If $f_1 \in R(\alpha)$ and $f_2 \in R(\alpha)$ on $[a,b]$ then _________.

   [a] $f_1 \notin R(\alpha)$ [b] $f_1 + f_2 \in R(\alpha)$

   [c] $f_1 + f_2 \notin R(\alpha)$ [d] $f_1 - f_2 \notin R(\alpha)$

3. The limit function of the series of the continuous functions need not be __

   [a] discontinuous [b] continuous

   [c] limit function [d] bounded

4. If $\{f_n\}$ is a sequence of continuous functions on E and if $f_n \to f$ uniformly on E then f is _____ on E

   [a] continuous [b] discontinuous

   [c] converges [d] diverges

5. There exists a real ______ function on the real line which is nowhere differentiable.

[a] compact | [b] complete
[c] differentiable | [d] continuous

6. Every number of an equicontinuous family is

[a] continuous | [b] discontinuous
[c] equicontinuous | [d] uniform continuous

7. If $0 < t < 2\pi$ then $E(it) \neq$ _____.

[a] 1 | [b] 0
[c] e | [d] π

8. If z is complex number with $|z| = 1$ there is a unique t in [0,2 π]such that

[a] $E(it) = e$ | [b] $E(it) = 2\pi$
[c] $E(it) = z$ | [d] $E(it) = 0$

9. Let $\{\phi(n)\}(n = 1, 2, 3, ...)$ be a sequence of complex functions on [a,b] then $\{\phi(n)\}$ is said to an ______system of functions on [a,b]

[a] orthogonal | [b] orthonormal
[c] normal | [d] sequence function

10. If $f(x) = 0$ for all x in some segment J then $S_N(f : X) =$ ____for every $x \in J$.

[a] 1 | [b] -1
[c] 0 | [d] ∞

SECTION – B [5 X 7 = 35]

Answer ALL the Questions.

11. a) If P^* is a refinement P, then prove that

i) $L(P, f, \alpha) \le L(P^*, f, \alpha)$

ii) $U(P^*, f, \alpha) \le U(P, f, \alpha)$

[OR]

b) State and prove fundamental theorem of calculus.

12. a) Suppose $f_n \to f$ uniformly on a set E in a metric space. Let x be a limit point of E and suppose that $\lim_{t \to \infty} f_n(t) = A_n \, (n = 1, 2, 3, ...)$ then prove that $\{A_n\}$ converges and $\lim_{x \to \infty} f_n(t) = \lim_{n \to \infty} A_n$.

[OR]

b) Suppose K is compact and

i) $\{f_n\}$ is a sequence of continuous functions on K.

ii) $\{f_n\}$ converges pointwise to a continuous function f on K.

iii) $f_n(x) \ge f_{n+1}(x)$ for all $x \in K, n = 1, 2, 3, ...$ then prove that $fn \to f$ uniformly on K.

13. a) If $\{f_n\}$ is a pointwise bounded sequence of complex functions on a countable set E, then prove that $\{f_n\}$ has a subsequence $\{f_{nk}\}$ such that $\{f_{nk}(x)\}$ converges for every $x \in E$.

[OR]

b) If K is compact, if $f_n \in \zeta(K)$ for $n = 1, 2, 3, ...$ and if $\{f_n\}$ is pointwise bounded and equicontinuous on K, then prove that

i) $\{f_n\}$ I uniformly bounded on K

ii) $\{f_n\}$ contains a uniformly convergent subsequence.

14. a) Prove that there exists a real continuous function on the real line which is nowhere differentiable

[OR]

b) State and prove Taylor's theorem

15. a) If for some x, there are constants $\delta > 0$ and $M < \infty$ such that $| f(x+t) - f(x) | \leq M | t |$ for all $t \in (-\delta, \delta)$ then prove that $\lim_{N \to \infty} S_N(f; x) = f(x)$.

[OR]

b) If $x > 0$ and $y > 0$ then prove that $\int_0^1 t^{x-1}(1-t)^{y-1} dt = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$.

SECTION – C **[3 X 10 = 30]**

Answer Any THREE Questions.

16. If γ' is continuous on $[a, b]$, then prove that γ is rectifiable and

$$\Lambda(\gamma) = \int_a^b |\gamma'(t)| dt.$$

17. Prove that $\int_0^1 [\lim_{n \to \infty} f_n(x)] dx = 0$ when

$$f_n(x) = n^2 x(1-x^2)^n (0 \leq x \leq 1, n = 1, 2, 3....)$$

18. Stand and prove Stone Weierstrass theorem.

19. Suppose $\sum C_n$ converges. Put $f(x) = \sum_{n=0}^{\infty} C_n x^n$, then prove that

$$\lim_{x \to 1} f(x) = \sum_{n=0}^{\infty} C_n$$

20. State and prove parseval's theorem.

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END SEMESTER EXAMINATION - NOVEMBER 2019

Programme : M.Sc. Mathematics
Course Code : 17PMAC23
Course Title : Partial Differential Equations

Date : 18.11.2019
Time : 2.00p.m. to 5.00p.m.
Max Marks :75

SECTION – A **[10 X 1 = 10]**

Answer ALL the Questions.

Choose the Best Answer.

1. $x^2p + yq = (x-y)z^2 + x - y$ is __________.

   [a] quasi linear [b] semi linear

   [c] linear [d] none of these

2. The partial differential equation of the form $f(x,y,z)\left(\frac{\partial z}{\partial x}\right) + g(x,y,z)\left(\frac{\partial z}{\partial y}\right) = h(x,y,z)$ is called ______________.

   [a] linear equation [b] semi linear equation

   [c] non linear [d] quasi linear equation

3. The complete integral of $q = 3p^2$ is _________.

   [a] $z = ax + 3a^2y + b$ [b] $z = ay + 3b^2x + c$

   [c] $z^2 = 6ay + a$ [d] $2z = 2ax + 6a^2y + 3b$

4. The complete integral of $z = pq$ is _________.

   [a] $z = (x+b)(x+a)$ [b] $z^2 = (x+a)$

   [c] $z = ab$ [d] $z = a + b$

5. Along every characteristic strip of the partial differential equation $f(x, y, z, p, q) = 0$ the function $f(x, y, z, p, q)$ is __________.

[a] independent [b] dependent

[c] constant [d] infinite

6. $\int \left(\frac{1}{p^3} dp_3 + \frac{1}{x^3}\right) = 0$ is _____________.

[a] $p_3 x_3 = c$ [b] $p_3 + x_3 = c$

[c] $log p_3 c_3 = x_3$ [d] $\frac{p_3}{x_3} = c_1$

7. Integrating partially with respect to y, once $\frac{1}{D'}(x^4 y^5)$.

[a] $\frac{x^5 y^6}{6}$ [b] $x^4 y^6$

[c] $\frac{x^4 y^6}{6}$ [d] $\frac{x^4 y^5}{20}$

8. $\int e^{ax} \sin bx \, dx =$_________.

[a] $\frac{e^{ax}}{a^2+b^2}(a sinbx - b cosbx)$ [b] $\frac{e^{ax}}{a^2+b^2}(a \, cosbx + b sinbx)$

[c] $\frac{e^{ax}}{a^2-b^2}(a sinax + b cosax)$ [d] $\frac{e^{ax}}{a^2-b^2}(b cosax - b \, sinax)$

9. The partial differential equation $\left(\frac{\partial z}{\partial x}\right)\left(\frac{\partial z}{\partial y}\right) = 3xy$ is _____________.

[a] linear equation [b] non linear equation

[c] homogeneous equation [d] non homogeneous equation

10. The order and degree of the non linear partial differential equation

$$z^2 \left\{ \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 1 \right\} = 1.$$

[a] order-1; degree- 2 [b] order-2; degree- 1

[c] order-2; degree- 2 [d] order-1; degree- 1

SECTION – B **[5 X 7 = 35]**

Answer ALL the Questions.

11. a) Solve $a(p + q) = z$.

[OR]

b) Solve $p + 3q = 5z + tan(y - 3x)$.

12. a) Show that the equations $xp = yq$ and $z(xp + yq) = 2xy$ are compatible and solve them.

[OR]

b) Find a complete integral of $px + qy = pq$.

13. a) Find a complete integral of ${p_1}^3 + {p_2}^2 + p_3 = 1$.

[OR]

b) Find a complete integral of ${x_3}^2 {p_1}^2 {p_2}^2 {p_3}^2 + {p_1}^2 {p_2}^2 - {p_3}^2 = 0$.

14. a) $(D^2 + 3DD' + 2D'^2)z = x + y$.

[OR]

b) $(2D^2 - 5DD' + 2D'^2)z = 24(y - x)$.

15. a) Solve $(x^2 D^2 + 2xyDD' + y^2 D'^2)z = x^2 y^2$.

[OR]

b) Solve $(x^2 D^2 - 4xyDD' + 4y^2 D'^2 + 4yD' + xD)z = x^2 y$.

SECTION – C **[3 X 10 = 30]**

Answer Any THREE Questions.

16. Solve $(y + z)p + (z + x)q = x + y$.

17. Find a complete integral of $z^2 = pqxy$.

18. Solve $p^2 x + q^2 y = z$ by Jacobi's method.

19. Solve $(D^2 - 3DD' + 2D'^2)z = e^{2x-y} + e^{x+y} + \cos(x + 2y)$.

20. Solve $(x^2 D^2 - 2xyDD' + y^2 D'^2 - xD + 3yD')z = {}^{8y}/_{x}$.

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END SEMESTER EXAMINATION - NOVEMBER 2019

Programme : M.Sc. Mathematics | **Date : 20.11.2019**
Course Code: 17PMAC24 | **Time : 2.00p.m. to 5.00 p.m.**
Course Title : Operations Research | **Max. Marks :75**

SECTION – A **[10 X 1 = 10]**

Answer ALL the Questions.

Choose the Best Answer.

1. In standard form II the initial identity matrix is obtained after introducing ___________ only.

   [a] Basic variables [b] slack variables

   [c] artificial variables [d] surplus variables.

2. ____________ variable is not required in the dual simplex method over the usual simplex method.

   [a] artificial [b] Revised Simplex

   [c] Dual Simplex [d]a or b

3. The slack for an activity is equal to _________.

   [a] LF-LS [b] EF-ES [c] LS-ES [d]LS-EF

4. Latest start time of an activity in CPM is the

   [a] latest occurrence time of the successor event

   [b] satisfy precedence requirements

   [c] earliest occurrence time for the predecessor event

   [d] avoid use of resources.

5. Each of the principal minor determinants is ____________ for positive semidefinite.

   [a] Positive [b] negative

   [c]Positive or zero [d] negative or zero

6. Each of the principal minor determinants is __________ for negative finite.

   [a] Positive [b] negative

   [c]Positive or zero [d] negative or zero

7. In general quadratic programming problem if the function $X^T\ Q\ X$ definite then it is ____________ in X over all of R^nn.

   [a]concave [b] convex

   [c] a and b [d] a or b

8. In quadratic programming the objective function should be __________.

   [a] quadratic [b] linear

   [c] cubic [d] a or b

9. In which N.L.P.P the problem of minimizing a convex objective function in the convex set of point is called __________ programming.

   [a] convex [b] concave

   [c] a or b [d] a and b

10. Two separable programming problem after getting the resulting linear programming problem is to be solved by ____________ method.

    [a] Two phase [b] dual simplex

    [c] simplex [d]big-M

SECTION – B [5 X 7 = 35]

Answer ALL the Questions.

11. a) Solve the following simple linear programming problem by revised simplex method max $z = x_1 + 2x_2,$

subject to $x_1 + x_2 \leq 3, x_1 + 2x_2 \leq 5, 3x_1 + x_2 \leq 6$ and $x_1, x_2 \geq 0$

[OR]

b) Use dual simplex method solve min $z = 3x_1 + x_2,$

subject to $x_1 + x_2 \geq 1, 2x_1 + 3x_2 \geq 2,$ and x_1 *and* $x_2 \geq 0$

12. a) Explain network diagram representation.

[OR]

b) A project consists of a tasks labeled A,B,…,H,I with the following relationships(W<X,Y, means X and Y cannot start until W is complete, X,Y<W means W cannot start until both X and Y are complete), Construct the network diagram having the following condition :A<D,E,B,D<F,C<G,B<H,F,G<I. Find also the optimum time of the project, when the time in days co completion of each task is as follows:

Task	A	B	C	D	E	F	G	H	I
Time	23	8	20	16	24	18	19	4	10

13. a) Prove that A sufficient condition for a stationary point x_0 to be an extreme point is that the hessian matrix H evaluated at x_0 is, (i) negative definite when x_0 is a maximum point

[OR]

b) Find the maximum or minimum of the function

$$f(X) = x_1^2 + x_2^2 + x_3^2 - 4x_1 - 8x_2 - 12x_3 + 56$$

14. a) Explain Wolfe's modified simplex method.

[OR]

b) Apply Wolfe's method for solving the quadratic programming problem:

$max\ Z_x = 2x_1 + x_2 - x_1^2$, subject to $2x_1 + x_2 \le 4$, $2x_1 + x_2 \le 4$ and $x_1, x_2 \ge 0$

.

15. a) Describe Separable function and reducible to separable form.

[OR]

b) Describe Reduction of separable programming problem to L.P.P.

SECTION – C **[3 X 10 = 30]**

Answer Any THREE Questions.

16. Solve the following problem by dual simplex method: $\min z = 2x_1 + x_2$, subject to $3x_1 + x_2 \ge 3$, $4x_1 + 3x_2 \ge 6$, $x_1 + 2x_2 \ge 3$, and

$x_1 \ge 2, x_2 \ge 0$,

17. The following table give the activities in a construction project and other relevant information

(i) Draw the activity network of the project.

(ii) Find the total float and free float for each activity

(iii) using the above information Crash or shorten the activity step by step until the shortest duration is reached.

activity	Preceding activity	Normal time days	Crash Time (days)	Normal Cost (Rs)	Crash cost (Rs)
(1-2)	--	20	17	600	720
(1-3)	--	25	25	200	200
(2-3)	(1-2)	10	8	300	440
(2-4)	(1-2)	12	6	400	700
(3-4)	(1-3),(2-3)	5	2	300	420
(4-5)	(2-4),(3,4)	10	5	300	600

18. Use the Kuhn-tucker condition to solve the following NLP problem

$$max\ z = 2x_1 - x_1^2 + x_2,$$

subject to $2x_1 + 3x_2 \le 6$, $2x_1 + x_2 \le 4$, and $x_1, x_2 \ge 0$,

19. Use Beale's method for solving the quadratic programming problem:

$\max Z_x = 4x_1 + 6x_2 - 2x_1^2 - 2x_1x_2 - 2x_2^2$, subject to

$x_1 + 2x_2 \le 2\ and\ x_1, x_2 \ge 0$

20. Use separable programming algorithm to solve the NLP problem

$\max z = 3x_1 + 2x_2$ subject to $4x_1^2 + x_2^2 \le 16$, $x_1 \ge 0, x_2 \ge 0$

b) Let $f_n(x)$ denote the distance from the real number x to the nearest number of the form $m/10^n$ where m, n are non negative integers and $x \in (0,1)$. Show that $f = \sum_{n=1}^{\infty} f_n$ is continuous and is differentiable nowhere on $(0,1)$.

15. a) Prove that a function $f \in BV[a,b]$ if and only if f is the difference of two finite-valued monotone increasing functions on $[a,b]$, where a and b are finite.

[OR]

b) If f is a finite valued monotone increasing function defined on the finite interval $[a,b]$, then prove that f' is measurable and $\int_a^b f'\,dx \leq f(b) - f(a)$.

SECTION – C [3 X 10 = 30]

Answer Any THREE Questions.

16. Prove that the outer measure of an interval equals its length.
17. Let c be any real number and let f and g be real valued measurable functions defined on the same measurable set E. Prove that $f+c, cf, f+g, f-g$ and fg are measurable.
18. (i) Show that lim $\int_0^{\infty} \frac{dx}{(1+x/n)^n x^{1/n}} = 1$.

    (ii) Show that $\int_0^{\infty} \frac{\sin t}{e^t - x} dt = \sum_{n=1}^{\infty} \frac{x^{n-1}}{n^2+1}, -1 \leq x \leq 1$.
19. Let f be a bounded function defined on the finite interval $[a,b]$. Prove that f is Riemann integrable over $[a,b]$ if and only if, it is continuous a.e.
20. Let $[a,b]$ be a finite interval and let $f \in L\,(a,b)$ with indefinite integral F, prove that $F' = f\,a.e.\,in\,[a,b]$.

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END SEMESTER EXAMINATION - NOVEMBER 2019

Programme : M. Sc., Mathematics **Date:14.11.2019**

Course Code: 17PMAC31 **Time: 10.00a.m. to 1.00p.m.**

Course Title : Measure Theory **Max. Marks :75**

SECTION – A [10 X 1 = 10]

Answer ALL the Questions.

Choose the Correct Answer.

1. If $A \subseteq B$ then ______.

   [a] $m^*(A) \leq m^*(B)$ [b]$m^*(A) \geq m^*(B)$

   [c]$m^*(A) = 0$ [d] None of these.
2. $m^*([1.2]) =$ _______

   [a] 2 [b] 1

   [c] 0 [d] 3
3. Ess sup $f =$ _____.

   [a]$Ess\ inf(-f)$ [b]$-Ess\ inf(-f)$

   [c]$Ess\ inf(f)$ [d]$-Ess\ sup(-f)$
4. Let f be a measurable function and A be a Borel set. Then $f^{-1}(A)$ is a ______.

   [a] measurable set [b] countable set

   [c] empty set [d] non measurable set

5. If f is a non-negative measurable function f, then $f = 0\ a.e\ \ iff$ ______.

[a]$f = 0$ [b]$\int f dx = 0$

[c]$\int f dx \neq 0$ [d]$\int f dx < \infty$

6. If f is a measurable function such that atleast one of $\int f^+ dx, \int f^- dx$ is finite, then $\int f dx =$ _____.

[a] $\int f^+ dx + \int f^- dx$ [b]$\int f^+ dx - \int f^- dx$

[c]$\int f^+ dx$ [d] 0

7. Let f be a bounded measurable function defined on the finite interval $(a.b)$. Then $\lim_{\beta\to\infty} \int_a^b f(x) sin\beta x\ dx =$ _____.

[a]$b - a$ [b]$a + b$

[c] 0 [d]$a - b$

8. Which one of the following is true?

[a]$D^+(-f) = -D_+(f)$ [b]$D^+(f) = D_+(-f)$

[c]$D^+(f) = -D_+(-f)$ [d]$D^+(-f) > -D_+(f)$

9. B$V[a, b]$ is a vector space over ______.

[a] the rationales [b] the real numbers

[c] the integers [d] the complex numbers

10. If $f\ \in BV[a, b]$ where a and b are finite then ______.

[a]f is differentiable [b] f is differentiable a.e

[c] f is integrable [d] none of these.

SECTION – B **[5 X 7 = 35]**

Answer ALL the Questions.

11. a) Prove that every interval is measurable.

[OR]

b) Prove that the following statements regarding the set E are equivalent:

(i) E is measurable.

(ii) $\forall \epsilon > 0, \exists O$ an open set, $O \supseteq E$ such that $m^*(O - E) < \epsilon$

(iii) $\exists G$, a G_δ-set, $G \supseteq E$ such that $m^*(G - E) = 0$.

12. a) Prove that the following statements are equivalent:

(i) f is a measurable function.

(ii) $\forall \alpha, \{x: f(x) \geq \alpha\}$ is measurable.

(iii)$\forall \alpha, \{x: f(x) < \alpha\}$ is measurable.

(iv)$\forall \alpha, \{x: f(x) \leq \alpha\}$ is measurable.

[OR]

b) Let T be a measurable set of positive measure and let $\mathbf{T}^* = \{\mathbf{x} - \mathbf{y}: \mathbf{x} \in \mathbf{T}, \mathbf{y} \in \mathbf{T}\}$. Show that $\mathbf{T}^*$ contains an interval $(-\propto, \propto)$ for some $\propto > 0$.

13. a) Let f and g be non-negative measurable functions. Prove that $\int f dx + \int g dx = \int (f + g) dx$.

[OR]

b) State and prove the Lebesgue's Dominated Convergence Theorem.

14. a) If f is Riemann integrable and bounded over the finite interval $[a, b]$ then prove that f is integrable and $R \int_a^b f dx = \int_a^b f dx$.

[OR]

SECTION – C **[3 X 10 = 30]**

Answer Any THREE Questions.

16. Establish the characterisation of a topological space in terms of a closure operator.
17. i) Show that the relation of homeomorphism on the set of all topological spaces is an equivalence relation.

    ii) Consider

    $X = \{a,b,c\}, Y = \{p,q,r\}$

    $\tau_1 = \{\phi, X, \{a,b\}, \{c\}\}$

    $\tau_2 = \{\phi, X, \{p\}, \{q\}, \{r\}, \{p,q\}, \{p,r\}, \{r,q\}\}$

    Is (x,τ_1) and (y,τ_2) are homeomorphic? Justify.
18. Let (X,τ) be the topological space. Then prove that

    (a) Each point in X contained in exactly one component of X.

    (b) The components of X form a partition of X.

    (c) Each connected subset of X contained in a component of X.

    (d) Each connected subset of X which is both open and closed is a component of X.
19. Prove that a topological space (X,τ) is compact if and if every collection of $\tau-$closed subsets of X with finite intersection properly has a non-empty intersection.
20. Prove that (i) the product space $X = \Pi\{X_\alpha : \alpha \in \Lambda\}$ is T_1 if and only if each co-ordinate space is T_1.

    (ii) the product space $X = \Pi\{X_\alpha : \alpha \in \Lambda\}$ is T_2 if and only if each co-ordinate space is T_2.

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END SEMESTER EXAMINATION - NOVEMBER 2019

Programme :M. Sc., Mathematics	**Date: 16.11.2019**
Course Code:17PMAC32	**Time: 10.00 a.m. to 1.00 p.m.**
Course Title : Topology	**Max Marks :75**

SECTION – A **[10 X 1 = 10]**

Answer ALL the Questions.

Choose the Correct Answer.

1. In a _________ space, every subset is either open or closed.

   [a]discrete [b] indiscrete

   [c] door [d] countable
2. Which one of the following is not Hausdorff?

   [a] Discrete space [b] Co-finite topology on an infinite set

   [c] (R, U) [d] (R, S)
3. A homeomorphic image of a second countable space is ________.

   [a] first countable [b] second countable

   [c] open [d] closed
4. The relation of homeomorphism on the set of all topological spaces is an ___________.

   [a] equivalence relation [b] bijection

   [c] first countable [d] second countable

5. Every component of a topological space is __________.

   [a] open [b] clopen

   [c] closed [d] dense

6. The closure of a connected set is ________.

   [a] component [b] locally connected

   [c] dense [d] connected

7. Ever compact topological space has _________.

   [a] Bolzano-weirestrass property [b] sequentially compact set

   [c] locally compact subspace [d] finite intersection property

8. Which one of the following is an example of a compact space?

   [a] Co-finite topology [b] Infinite discrete topology

   [c] Hausdorff space [d] Connected space

9. Each projection map on a product space is __________.

   [a] closed [b] open

   [c] homeomorphism [d] bijective

10. The product space of two first countable topological space is ________.

   [a] second countable [b] first countable

   [c] Hausdorff [d] T_1-space

SECTION – B **[5 X 7 = 35]**

Answer ALL the Questions.

11. a) Prove that intersection of two topologies is a topology. Is union of two topologies, a topology? Justify your answer.

[OR]

b) Let (x, y) be a topological space and $A \subseteq X$. Prove that $\overline{A} = A \cup D(A)$.

12. a) Derive the characterization of continuous function in terms of open and closed sets.

[OR]

b) Derive the criteria for open mapping in terms of interior.

13. a) Prove that the union of any family of connected sets having a non-empty intersection is a connected set.

[OR]

b) Prove that a subset E of a real line R containing atleast two points is connected if and only if A is an interval.

14. a) Prove that every closed subspace of a compact space is compact.

[OR]

b) Let (x, τ) be a connected Hausdorff space. Show that no non-empty open proper subset of X is compact.

15. a) Derive the characterisation of a topological space in terms of a base.

[OR]

b) Let X and Y be the topological spaces. Prove that $X \times Y$ is connected if and only if X and Y are connected.

[OR]

b) Prove that a spherical helix projects on a plane perpendicular to its axis in an arc of an epicycloid.

15. a) Calculate the first fundamental coefficients and the area of the anchor ring corresponding to the domain $0 \leq u \leq 2\pi$ and $0 \leq v \leq 2\pi$.

[OR]

b) Prove that the curves of the family $\frac{v^3}{u^2} =$ constant are geodesics on a surface with a metric $v^2du^2 - 2uvdudv + 2u^2dv^2, u > 0, v > 0$.

SECTION – C **[3 X 10 = 30]**

Answer Any THREE Questions.

16. Find the arc length of a curve between two points.
17. Calculate the torsion and curvature of the cubic curve $r = (u, u^2, u^3)$.
18. Find the curvature and torsion of the curve of intersection of the quadratic surfaces $ax^2 + by^2 + cz^2 = 1, a'x^2 + b'y^2 + c'z^2 = 1$.
19. State and prove the fundamental existence theorem for space curves.
20. (i) When $v = c$ for all values of u, prove that a necessary and sufficient condition that the curve $v = c$ is a geodesic is $EE_2 + FE_1 - 2EF_1 = 0$.

    (ii) When $u = c$ for all values of v, prove that a necessary and sufficient condition that the curve $u = c$ is a geodesic is $GG_1 + FG_2 - 2GF_2 = 0$.

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END SEMESTER EXAMINATION - NOVEMBER 2019

Programme : M. Sc., Mathematics **Date: 19.11.2019**

Course Code: 17PMAC33 **Time: 10.00a.m. to 1.00p.m.**

Course Title : Differential Geometry **Max. Marks :75**

SECTION – A **[10 X 1 = 10]**

Answer ALL the Questions.

Choose the Correct Answer.

1. Elimination of the parameter u in $x = u, y = u^2, z = u^3$ gives _______.

   [a] $y = x^2 and\ xz = y^2$ [b] $x + y = 0$

   [c] $x^2 + z^2 = y^2$ [d] none of these

2. A real valued function f defined on real interval I is said to be of class m, m is positive integer, and if f has continuous mth derivative at every point of I, f is called as _________.

   [a]C^∞ function [b] C^m function

   [c] C^ω function [d] none of these

3. The necessary and sufficient condition for a curve to be a plane is that

   [a] $\tau = 0$ at all points [b] $k = 0$ at all points

   [c] $\tau \neq 0$ at all points [d] $k \neq 0$ at all points

4. The curvature of the circular helix is _______.

[a] $\frac{a^2+b^2}{a}$ [b] $\frac{a}{a^2+b^2}$

[c] $\frac{b}{a^2+b^2}$ [d] $\frac{a^2+b^2}{b}$

5. The radius of spherical curvature of a circular helix is equal to _______.

[a] radius of curvature [b] centre of curvature

[c] torsion [d] none of these

6. Let δ be a curve $r(u)$, and let S be a surface $F(x,y,z)=0$. If $F'(u_0)\neq 0$, u_0 is a simple zero of $F(u)=0$, then the curve δ and the surface 'S' is _______.

[a]Two point contact [b]Three point contact

[c] Simple intersection at $r(u_0)$ [d]n point contact

7. A necessary and sufficient condition for a curve to be helix is that the ratio of the curvature to torsion is _____________

[a]constant at all points [b]zero at all points

[c]constant at all points [d]zero at only one points

8. A space curve lying on a cylinder and cutting the generators of the cylinder at a constant angle is called ______________

[a]circular helix [b]cylindrical helix

[c]spherical helix [d]none of these

9. The value of H for the paraboloid $x=u, y=v, z=u^2-v^2$ is _______

[a] $\sqrt{4u^2+4v^2+1}$ [b] $1+4u^2$

[c] $1+4v^2$ [d]-4uv

10. The unit normal vector $\vec{N}$ _______________.

[a] $r_1\times r_2$ [b] $1+(r_1\times r_2)$

[c] $\frac{H}{r_1\times r_2}$ [d] $\frac{r_1\times r_2}{H}$

SECTION – B [5 X 7 = 35]

Answer ALL the Questions.

11. a) Write the two-equivalent representation of circular helix.

[OR]

b) Find the arc length of one complete turn of the circular helix $r(u)=(a\cos u, a\sin u, bu), -\infty<u<\infty$

12. a) Find the directions and equations of the tangent, normal and binormal and also obtain the normal, rectifying and osculating planes at a point on the circular helix $r=(a\cos\left(\frac{s}{c}\right), a\sin\left(\frac{s}{c}\right), b\left(\frac{s}{c}\right))$.

[OR]

b) Prove that the necessary and sufficient condition for a curve to be a straight line is that $k=0$ at all points of the curve.

13. a) Find the centre and radius of the spherical curvature of the curve $r=r(s)$ at a point P on the curve γ.

[OR]

b) Show the necessary and sufficient condition that a curve lies on a sphere is $\frac{\rho}{\sigma}+\frac{d}{ds}(\sigma\rho')=0$.

14. a) With usual notation, prove that a necessary and sufficient condition for a curve to be helix is that the ratio of the curvature to torsion is constant at all points.

14. a) Let G be a bipartite graph with bipartition (X, Y). Then prove that G contains a matching that saturates every vertex in X if and only if $|N(S)| \geq |S|$ for all $S \subseteq X$.

[OR]

b) Prove that in a bipartite graph, the number of edges in a maximum matching is equal to the number of vertices in a minimum covering.

15. a) Let G be a connected graph that is not an odd cycle. The prove that G has a 2-edge colouring in which both colours are represented at each vertex of degree at least two.

[OR]

b) Prove that if G is a bipartite, then $\chi' = \Delta$.

SECTION – C **[3 X 10 = 30]**

Answer Any THREE Questions.

16. Prove that $\sum_{\upsilon \in V} d(\upsilon) = 2 \in$ and also a graph is bipartite if and only if it contains no odd cycle.

17. (i) Prove that if e is a link of G, then $\tau(G) = \tau(G-e) + \tau(G.e)$

(ii) State and prove Cayley's formula.

18. Explain about the Chinese postman problem and the Travelling Salesman problem.

19. Prove that G has a perfect matching if and only if $o(G-S) \leq |S|$ for all $S \subset V$.

20. State and prove Vizing's theorem.

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END SEMESTER EXAMINATION - NOVEMBER 2019

Programme: M. Sc. Mathematics **Date:21.11.2019**
Course Code: 17PMAC34 **Time: 10.00a.m. to 1.00p.m.**
Course Title : Graph Theory **Max Marks :75**

SECTION – A **10 X 1 = 10]**

Answer ALL the Questions.

Choose the Correct Answer.

1. The number of edges of a simple complete bipartite graph $K_{m,n}$ is ____

[a] $m+n$ [b] mn

[c] $\frac{m}{n}$ [d] $m-n$

2. From the following sequences which one is graphic _____

[a] (7,6,5,4,3,3,2) [b] (6,6,5,4,3,3,1)

[c] (7,6,6,4,4,3,3) [d] (2,2,2,2,2,2,2)

3. Every nontrivial tree has at least two vertices of degree

[a] one [b] two

[c] three [d] four

4. The number of spanning trees of K_6 is ________

[a] 36 [b] 216

[c] 1296 [d] 7776

5. The Herschel graph is ______

[a] Hamiltonian [b] nonhamiltonian

[c] Eulerian [d] not Eulerian

6. The sequence (2,2,2,2,2) is degree majorized by another sequence, which is _______

[a] (2,2,2,1,1) [b] (2,2,2,2,1)

[c] (2,2,2,2,0) [d] (2,2,2,3,3)

7. The Petersen graph is ________

[a] 1-factorable [b] 2-factorable

[c] 3-factorable [d] not 1-factorable

8. The number of perfect matchings in a tree is _____

[a] one [b] two

[c] atmost one [d] three

9. The edge chromatic number of $K_{9,10}$ is_____

[a] 9 [b] 10

[c] 19 [d] 1

10. This edge chromatic number of K_{2n} is _____

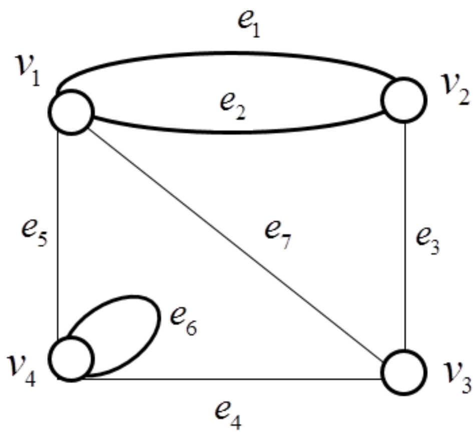
[a] $2n$ [b] $2n+1$

[c] $2n-1$ [d] $2(n-1)$

SECTION – B **[5 X 7 = 35]**

Answer ALL the Questions.

11. a) Find the incidence and adjacency matrices for the graph



[OR]

b) State and prove Sperner's lemma.

12. a) Prove that an edge e of G is a cur edge of G if and only if e is contained in no cycle of G.

[OR]

b) Prove that $K \leq K' \leq \delta$.

13. a) Prove that if G is a simple graph with $\nu \geq 3$ and $\delta \geq \frac{\nu}{2}$, then prove that G is hamiltonian.

[OR]

b) Define closer of a graph and prove that C(G) is well defined.

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END SEMESTER EXAMINATION - NOVEMBER 2019

Programme : M.Sc. Mathematics | **Date : 14.11.2019**
Course Code: 17PMAC41 | **Time : 2.00 p.m. to 5.00 p.m.**
Course Title : Complex Analysis | **Max. Marks :75**

SECTION – A [10 X 1 = 10]

Answer ALL the Questions.
Choose the Correct Answer.

1. If the function $f(z)$ is analytic at some point in every neighborhood of a point $z_0$function with $z_0$itself, then $z_0$is called an ____________singularity of $f(z)$.

   [a] isolated [b] removable

   [c] a and b [d] a or b

2. The function $f(z) = xy + iy$ is everywhere continuous but it is ________.

   [a] analytic [b] not analytic

   [c] differentiable [d] harmonic

3. The primitive period of $\cos z$is ______________.

   [a] 2π [b] $2\pi i$ [c] $\frac{\pi}{2}$ [d] $\frac{\pi i}{2}$

4. The primitive period of $\sin z$is ______________.

   [a] 2π [b] $2\pi i$ [c] $\frac{\pi}{2}$ [d] $\frac{\pi i}{2}$

5. Any two indefinite integral of a function differ by ____________.

[a] 0 [b] constant

[c] variable [d] 1

6. The smallest period of a real valued periodic function $f(x)$ is called the _________period of $f(x)$.

[a] derivative [b] primitive

[c] indefinite [d] definite

7. If a function $f(z) = e^{\frac{1}{z}}$ has an isolated essential singularity at z = _____.

[a] 0 [b] 1 [c] 3 [d] 4

8. Number of zeros of the function $f(z) = \sin\frac{1}{z}$ is _________.

[a] 2 [b] 4 [c] infinite [d] finite

9. The number of isolated singular points of $f(z) = \dfrac{z+3}{z^2(z^2+2)}$ is

[a] 1 [b] 2 [c] 3 [d] 4

10. The value of $\frac{1}{2\pi i}\int \frac{e^z}{z-2}\, dz$ is _________.

[a] 0 [b] 1 [c] e^2 [d] infinite

SECTION – B [5 X 7 = 35]

Answer ALL the Questions.

11. a) Show that the function $e^x(\cos y + i \sin y)$ is holomorphic and find its derivatives.

[OR]

b) Show that ananalytic function with constant modulus is constant.

12. a) State and prove Abel's limit theorem.

[OR]

b) State and prove Addition theorem for exponential function e^z.

13. a) Derive Cauchy's inequality.

[OR]

b) Evaluate $\int_C \frac{e^{2z}}{(z+1)^4}\, dz$ where the path of integration C is $|z| < 1$.

14. a) State and prove the Liouville's theorem.

[OR]

b) Find the Laurent's series of the function $f(z) = \frac{1}{(z^2-4)(z+1)}$valid in the region $1 \leq |z| \leq 2$.

15. a) State and prove the Schwarz lemma.

[OR]

b) Find the residues of the function $f(z) = \frac{1}{(z^2-4)(z+1)}$.

SECTION – C [3 X 10 = 30]

Answer Any THREE Questions.

16. If $f(z)$ is an analytic function of z, prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4\,|f'(z)|^2$.

17. Find the domains of convergence of the following series:

(i) $\sum_1^\infty \frac{1.3.5\ldots(2n-1)}{n!}\left(\frac{1-z}{z}\right)^n$ (ii) $\sum_2^\infty \frac{z^n}{n(\log n)^2}$.

18. Let $f(z)$ be analytic function within and on the boundary C of a simple connected region D and let z_0 be any point within C. Then prove that $f'(z) = \frac{1}{2\pi i}\int_C \frac{f(z)}{(z-z_0)^2} dz.$

19. Define all singularities of an analytic function with suitable examples.

20. State and prove the Alternative form of Schwarz lemma.

15. a) If T is normal then prove that M_i's are pairwise orthogonal.

[OR]

b) If T is normal then prove that the M_i's span H.

SECTION – C **[3 X 10 = 30]**

Answer Any THREE Questions.

16. State and prove the Hahn – Banach theorem.
17. State and prove the open mapping theorem.
18. Let H be a Hilbert space and let f be an arbitrary functional in H^* . Then prove that there exists a unique vector y in H such that f(x) = <x, y>.
19. i) If N_1 and N_2 are normal operators on H with the property that either commutes with the adjoint of the other, then prove that $N_1 + N_2$ and N_1N_2 are normal.

    ii) If T is an operator on H then prove that T is normal $\Leftrightarrow$ its real and imaginary parts commute.
20. Prove that two matrices in A_n are similar iff they are the matrices of a single operator on H relative to different bases.

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END SEMESTER EXAMINATION - NOVEMBER 2019

Programme :M. Sc., Mathematics	**Date : 16.11.2019**
Course Code: 17PMAC42	**Time: 2.00p.m. to 5.00p.m.**
Course Title : Functional Analysis	**Max Marks :75**

SECTION – A **[10 X 1 = 10]**

Answer ALL the Questions.

Choose the Correct Answer.

1. $\|\alpha x\| =$______ (α is a scalar)

   [a] $\alpha\|x\|$ [b] $|\alpha|\|x\|$

   [c] $\|\alpha\|\|x\|$ [d] $|\alpha\|x|$

2. $\|x\| - \|y\| \le$______.

   [a] $|x-y|$ [b] -$|x-y|$

   [c] $\|x-y\|$ [d] $|x+y|$

3. If B is a reflexive Banach space then its closed unit sphere S is ______.

   [a] compact [b] connected

   [c] complete [d] weakly compact

4. A _______ on a Banach space B is an idempotent operator on B.

   [a] compact [b] complete space

   [c] projection [d] connected

5. In a Hilbert space H $\|x+y\|^2+\|x-y\|^2=2\|x\|^2+2\|y\|^2$ is known as

[a] Bessel's inequality [b] Schwarz inequality

[c] Parallelogram law [d] none of the above

6. If S is a non-empty subset of a Hilbert space then $S^{\perp}$ =___

[a] S [b] $S^{\perp\perp}$

[c] $S^{\perp\perp\perp}$ [d] - $S^{\perp}$

7. The adjoint operation on $B(H)$, $(T_1 T_2)^*$ = ______.

[a] T_1T_2 [b] $T_2^* T_1^*$

[c] $T_1^* T_2^*$ [d] T_2T_1

8. If N is a normal operator on H then $\|N^2\|$= ______

[a] $\|N\|^2$ [b] - $\|N\|^2$

[c] $2\|N\|$ [d] - $\|N^2\|$

9. The dimension of $B(H)$ is ______.

[a] n [b] 2n

[c] 3n [d] n^2

10. Let T be an operator on H. Then T is singular iff _____.

[a] $1\in\sigma(T)$ [b] $-1\in\sigma(T)$

[c] $0\in\sigma(T)$ [d] $e\in\sigma(T)$

SECTION – B [5 X 7 = 35]

Answer ALL the Questions.

11. a) Let N and N' be normed linear spaces then prove that the set B(N, N') of all continuous linear transformations N into N' is itself a normed linear space with norm $\|T\|=\sup\{\|Tx\|/\|x\le 1\|\}$.

[OR]

b) If N is a normed linear space and x_0 is non-zero vector in N then prove that there exist a functional f_0 in N^* such that $f_0(x_0)=\|x_0\|$ and $\|f_0\|$=1.

12. a) State and prove the closed graph theorem.

[OR]

b) State and prove the uniform boundedness theorem.

13. a) Prove that a closed convex subset C of a Hilbert space H contains a unique vector of smallest norm.

[OR]

b) If M and N are closed linear subspaces of Hilbert space H such that $M\perp N$, then prove that the linear subspace M + N is also closed.

14. a) Prove that in the adjoint operation $T\to T^*$ on B(H) has the following properties: a) $(T_1+T_2)^*=T_1^*+T_2^*$ b) $T^{**}=T$ c) $\|T\|=\|T^*\|$

[OR]

b) If T is an operator on H for which <Tx, x> = 0 for all x, then prove that T = 0.

b) Find the eigen values of the homogenous integral equation

$$y(x) = \lambda \int_1^2 \left(xt + \frac{1}{xt}\right) y(t)dt.$$

14. a) Solve $y(x) = cosx + \lambda \int_0^{\pi} sin(x-t)y(t)dt.$

[OR]

b) Solve the Fredholm integral equation of the second kind

$$y(x) = x + \lambda \int_0^1 (xt^2 - x^2t)y(t)dt.$$

15. a) Solve the integral equation $y(x) = x + \lambda \int_0^1 xt\, y(t)dt$ by the method of successive approximations

[OR]

b) Solve $y(x) = f(x) + \frac{1}{2}\int_0^1 e^{x-t}y(t)dt$

SECTION – C **[3 X 10 = 30]**

Answer Any THREE Questions.

16. Show that the function y(x) = sin(π x /2) is a solution of the integral equation $y(x) - \frac{\pi^2}{4}\int_0^1 K(x,t)y(t)dt = \frac{x}{2}$ where $(x,t) = \begin{cases} \frac{x}{2}(2-t)\,, 0 \le x \le t \\ \frac{t}{2}(2-x),\ t \le x \le 1 \end{cases}$.
17. Obtain Fredholm integral equation of second kind corresponding to the boundary value problem $\frac{d^2\varphi}{dx^2} + \lambda\varphi = x, \varphi(0) = 0, \varphi(1) = 1$. Also, recover the boundary value problem from the integral equation obtained.
18. Determine the eigen values and eigen functions of the homogenous integral equation $y(x) = \lambda \int_0^1 K(x,t)y(t)dt$ where $K(x,t) = \begin{cases} -e^{-t}sinhx\,, 0 \le x \le t \\ -e^{-x}sinht, t \le x \le 1 \end{cases}$,
19. Solve y(x)= f(x)+λ$\int_0^1 (1-3xt)\ y(t)dt.$
20. Solve $y(x) = 1 - 2x - 4x^2 + \int_0^x [3 + 6(x-t) - 4(x-t)^2]\, y(t)\, dt$

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END SEMESTER EXAMINATION - NOVEMBER 2019

Programme : M. Sc., Mathematics **Date : 22.11.2019**
Course Code: 17PMAE11 **Time: 10.00a.m. to 1.00p.m.**
Course Title : Integral Equations **Max. Marks :75**

SECTION – A **[10 X 1 = 10]**

Answer ALL the Questions.

Choose the Correct Answer.

1. With usual notation the inner product of two functions *f* and g is defined as ________.

   [a] $\int_a^b f(x)\overline{g(x)}dx$ [b] $\int_a^b f(x)\overline{f(x)}dx$

   [c] $\int_a^b \frac{f(x)}{g(x)}dx$ [d] $\int_a^b \frac{\overline{g(x)}}{f(x)}dx$

2. Minkowski inequality is ________.

   [a] $|(f,g)| \le ||f||\ ||g||$ [b] $||f+g|| \le ||f|| + ||g||$

   [c] $||f+g|| \le ||f||||g||$ [d] $|f,g| \le ||f|| + ||g||$

3. Volterra integral equation of second kind for the initial value problem $y' - y = 0, y(0) = 1$ is __________.

   [a] $[u(x) = x + \int_0^x u(t)dt\ where\ u(x) = y'$

   [b] $u(x) = -x + \int_0^x u(t)dt\ where\ u(x) = y'$

   [c] $u(x) = 1 - \int_0^x u(t)dt\ where\ u(x) = y$

   [d] $u(x) = 1 + \int_0^x u(t)dt\ where\ u(x) = y'$

4. The initial value problem corresponding to the integral equation

$y(x) = 1 + \int_0^x y(t)dt$ is __________.

[a] y'-y=0, y(0)=1 [b] y'+y=0, y(0)=0

[c] y'-y=0, y(0)=0 [d] y'+y=0, y(0)=1

5. The kernel K(x,t)=(3x-t) t is ____________.

[a] symmetric and has an eigen function

[b] symmetric and has no eigen function

[c] not symmetric and has an eigen function

[d] not symmetric and has no eigen function

6. The integral equation $y(x) = \lambda \int_0^{2\pi} \sin(x+t)\, y(t)dt$ has __________

[a] two solutions for any value of λ

[b] infinitely many solutions for only one value of λ

[c] unique solution for every value of λ

[d] infinitely many solutions for two values of λ

7. The solution of the integral equation $g(s) = s + \int_0^1 su^2 g(u)du$ is _____.

[a] $g(t) = \frac{3t}{4}$ [b] $g(t) = \frac{4t}{3}$

[c] $g(t) = \frac{2t}{3}$ [d] $g(t) = \frac{3t}{2}$

8. When λ= 2, the equation $y(x) = f(x) + \lambda \int_0^1 (1-3xt)y(t)dt$ has ____.

[a] No solution [b] unbounded solution

[c] many solution [d] unique solution

9. The iterated kernels $K_n(x,t)$ of $K(x,t)=xe^t$, a=0, b=1 is __________.

$[a] e^{xt}$ [b] e^t

[c] xe^t [d] x

10. The general solution of $(D^2+1)h = 0$, where $D=\frac{d}{dt}$ is ___________.

$[a]\ h = A\cos t + B\sin t$ [b] $h = e^t(A\cos t + B\sin t)$

[c] $h = e^t$ [d] $e^{-t}(A\cos t + B\sin t)$

SECTION – B **[5 X 7 = 35]**

Answer ALL the Questions.

11. a) Show that the function y(x) = sin(2x) is a solution of the Fredholm integral equation $y(x) = \cos x + 3\int_0^{\pi} K(x,t)y(t)dt$ where

$$K(x,t) = \begin{cases} \sin x \cos t, & 0 \le x \le t \\ \cos x \sin t, & t \le x \le 1 \end{cases}$$

[OR]

b) Show that the function $y(x) = xe^x$ is a solution of the Volterra integral equation $y(x) = \sin x + 2\int_0^x \cos(x-t)y(t)dt$.

12. a) Convert the following initial value problem into an integral equation

$\frac{d^2y}{dx^2} + A(x)\left(\frac{dy}{dx}\right) + B(x)y = f(x)$, with the initial conditions

$y(a)=y_0$, $y'(a) = y_0'$.

[OR]

b) Derive the differential equation together with given initial conditions from integral equation

$$y(x) = 1 - x - 4\sin x + \int_0^x [3 - 2(x-t)]y(t)dt.$$

13. a) Solve the homogeneous Fredholm integral equation of the second kind

$y(x) = \lambda \int_0^{2\pi} \sin(x+t)y(t)dt.$

[OR]

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END SEMESTER EXAMINATION - NOVEMBER 2019

Programme :M. Sc. Mathematics **Date: 22.11.2019**

Course Code: 17PMAE21 **Time: 2.00 p.m. to 5.00 p.m.**

Course Title : Calculus of variations **Max. Marks :75**

SECTION – A **[10 X 1 = 10]**

Answer ALL the Questions.

Choose the Correct Answer.

1. A function $y = y(x)$ which extremizes a functional is called_______

   [a] Extremal [b] Functional

   [c] Curve [d] Variation

2. The shortest line between any two points on a cylinder is a ______

   [a] Circle [b] Straightline

   [c] Helix [d] Catenary

3. The extremals of the functional $I[y(x), z(x)] = \int_0^{\pi/2} (y^2 + z^2 + 2yz)dx$ are solutions of the simultaneous equations.

   [a] $y'' - z = 0, z'' - y = 0$ [b] $y'' + z = 0, z'' - y = 0$

   [c] $y'' - z = 0, z'' + y = 0$ [d] $y'' + z = 0, z'' + y = 0$

4. Extremal of the isometric problem $\int_{x_1}^{x_2} y'^2\,dx$ subject to $\int_{x_1}^{x_2} y\,dx = c$ is

[a] $y = \lambda x^2 + ax + b$ [b] $y = \lambda x^3 + ax + b$

[c] $y = ax + b$ [d] $y = \frac{\lambda x^2}{4} + ax + b$

5. The shortest distance between two points in a plane given by equation

[a] $(x-h)^2 + (y-k)^2 = r^2$ [b] $y = mx + c$

[c] $y = a^2 x + b$ [d] $y = x^3$

6. The distance between the curves $y_1(x) = x$ and $y_2(x) = x^2$ on the interval [0,1] is

[a] $\frac{1}{4}$ [b] $\frac{1}{2}$

[c] $\frac{3}{4}$ [d] 1

7. Extremal is maximum if $E \leq 0$ and extremal is minimum if $E \geq 0$ is _____

[a] Jacobi condition [b] Legendre condition

[c] Weistrass function [d] Hamilton's Principle

8. To embed an arc AB of the extremal in a central field of extremals, it is sufficient that the conjugate point of A does not lie on arc AB. This called

[a] Jacobi condition [b] Legendre condition

[c] Weistrass function [d] Hamilton's Principle

9. Find the eigen value of the problem $\frac{d^2y}{dx^2} = -\lambda y$ with $y(-1) = y(1) = 0$

[a] $\lambda = 2.5$ [b] $\lambda = 2$

[c] $\lambda = 3$ [d] $\lambda = 4$

10. Rayleigh-Ritz method is used to __________

[a] find maxima [b] find minima

[c] solve boundary value problems [d] find constant

SECTION – B **[5 X 7 = 35]**

Answer ALL the Questions.

11. a) Find the extremizing function for

$$J[z(x,y)] = \iint_D \left[\left(\frac{\partial^2 z}{\partial x^2}\right)^2 + \left(\frac{\partial^2 z}{\partial y^2}\right)^2 + 2\left(\frac{\partial^2 z}{\partial x \partial y}\right)^2 - 2zf(x,y) \right] dxdy \text{ where}$$

$f(x,y)$ is known function.

[OR]

b) Determine the extremal of the functional $I[y(x)] = \int_{-l}^{l} \left(\frac{1}{2} \mu y''^2 + \rho y \right) dx$,

subject to $y(-l) = 0, y'(-l) = 0, y(l) = 0, y'(l) = 0$.

12. a) Find the shortest path from the point $A(-2,3)$ to the point $B(2,3)$

located in the region $y \le x^2$.

[OR]

b) Find the function on which the following functional can be extremized

$I[y(x)] = \int_0^1 (y''^2 - 2xy)dx$, $y(0) = y'(0) = 0$. $y(1) = \frac{1}{120}$ and $y'(1)$ is not given.

13. a) Is the Jacobi condition fulfilled for the extremal of the functional

$I[y(x)] = \int_0^a (y'^2 + y^2 + x^2)dx$ passing through $A(0,0)$ and $B(a,0)$?

[OR]

b) Derive Legendre condition.

14. a) Find the shape of an absolutely flexible, inextensible homogeneous and heavy rope of given length l suspended at the points A and B.

[OR]

b) Discuss the isoperimetric problem.

15. a) Explain Rayleigh-Ritz method.

[OR]

b) Derive the Euler equation for the functional $I[y(x)] = \int_{x_1}^{x_2} F(x, y, y')dx$.

SECTION – C **[3 X 10 = 30]**

Answer Any THREE Questions.

16. Describe variational problems in parametric form.

17. Find the shortest distance between the parabola $y = x^2$ and the straight line $x - y = 5$.

18. Obtain the Weirstress function.

19. Derive the fundamental equation of quantum mechines from a variational principle.

20. Minimize $I[y] = \int_{-l}^{l}\left(\int_{-l}^{l} \frac{y'(s)}{x-s} ds\right) y(x)dx$ subject to

$J[y] = \int_{-l}^{l} y(x)dx = s =$ constant and the boundary conditions

$y(l) = y(-l) = 0$.

15. a) Let W_n denote a random variable with mean and variance b/n^p, *where* $p > 0$ μ *and* b are constant (not a fucntions of n). Prove that W_n converges stochastically to μ .

[OR]

b) Let $\overline{X}$ denote the mean of a random sample of size 100 from a distribution that is $\chi^2(50)$. Compute an approximate value of $P(49 < \overline{X} < 51)$.

SECTION – C **[3 X 10 = 30]**

Answer Any THREE Questions.

16. Let $f(x_1, x_2) = \begin{cases} 2x_1, & 0 < x_1 < 1, 0 < x_2 < 1 \\ 0, & elsewhere \end{cases}$ be the probability density function of x_1 and x_2.

Compute i) $E(X_1 + X_2)$ and

ii) $E[[X_1 + X_2 - X(X_1 + X_2)]^2]$.

17. Let $f(x, y) = \begin{cases} 2 & 0 < x < y < 1 \\ 0 & elsewhere \end{cases}$ be the joint p.d.f. of X and Y.

Show that the correlation coefficient between X and Y is 1/2.

18. Compute the measures of skewness and kurtosis of a gamma distribution with parameters α *and* β

19. Derive the F-distribution.

20. State and prove Central Limit theorem.

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END SEMESTER EXAMINATION - NOVEMBER 2019

Programme :M. Sc., Mathematics **Date: 19.11.2019**

Course Code:17PMAE41 **Time: 2.00p.m. to 5.00p.m.**

Course Title : Mathematical Statistics **Max. Marks :75**

SECTION – A **[10 X 1 = 10]**

Answer ALL the Questions.

Choose the Correct Answer.

1. Let $f(x) = \frac{1}{x^2}, 0 < x < \infty; 0$ *elsewhere* be the probability density function of X. If $A_1 = \{x : 1 < x < 2\}$. Then $P(A_1)$ = ___________.

   [a] 1 [b] ½

   [c] 2 [d] 0

2. $E(|X - b|)$ is minimum when 'b' is ____________.

   [a] median [b] mean

   [c] mode [d] maximum.

3. If X and Y are independent random variables, then $\rho =$ _______ .

   [a] 0 [b] 1

   [c] -1 [d] 2

4. The random variables X_1 and X_2 are said to be stochastically independent iff $f(x_1, x_2) \equiv$ _____________.

   [a] $f_1(x_1) . f_2(x_2)$ [b] $f_1(x_1)$

   [c] $f_2(x_2)$ [d] 0

5. If p = ½ , q = 2/3 and n = 5, then measure of skewness is __________.

[a] 0 [b] 1

[c] -1 [d] 5

6. In which distribution, the mean and variance are equal?

[a] Binomial [b] Poisson

[c] Normal [d] Gamma

7. Let X have the uniform distribution over the interval $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$. Then Y = tan X has a _____________ distribution.

[a] Chi square [b] Gamma

[c] Cauchy [d] Normal.

8. In a 't' distribution, the value of $\beta_2 =$_______.

[a] 0 [b] 3

[c] 1 [d] 2

9. In a limiting distribution, let U_n converge stochastically to c and let $P(U_n < 0) = 0$ for every n. Then the random variable $\sqrt{U_n}$ converges stochastically to _________.

[a] c [b] $\sqrt{c}$

[c] 0 [d] 1

10. If r = ½ , then the variance of chisquare distribution is ________,

[a] ½ [b] 2

[c] 1 [d] 0.

SECTION – B [5 X 7 = 35]

Answer ALL the Questions.

11. a) Let X be a continuous variable with space $\Re = \{x\,;\,0 < x < 1\}$. Let the probability set function be $P(A) = \int_A f(x)\,dx$ *where* $f(x) = c\,x^3, x \in \Re$. Find the constant 'c'.

[OR]

b) State and prove Chebyshev's inequality.

12. a) State and prove Baye's formula for conditional probability.

[OR]

b) Find Pr(0 < X_1 < 1/3, 0 < X_2 < 1/3) if the random variables X_1 and X_2 have the joint p.d.f. $f(x_1, x_2) = 4\,x_1\,(1 - x_2),\ 0 < x_1 < 1,\ \ 0 < x_2 < 1$, zero elsewhere.

13. a) Let X have a poisson distribution with $\mu = 100$. Use Chebyshev's inequality to determine a lower bound for $P(75 < X < 125)$.

[OR]

b) In a chi-square distribution, if $(1-2t)^{-6};\ t < \frac{1}{2}$ is the moment generating function of the random variable X, then find $P(X < 5.23)$.

14. a) Let T have a 't' distribution with 14 degrees of freedom. Determine 'b' so that $P(-b < T < b) = 0.90$.

[OR]

b) Let X and Y be random variables with $\mu_1 = 1,\ \mu_2 = 4$, $\sigma_1^2 = \sigma_2^2 = 6, \rho = 1/2$. Find the mean and variance of Z=3X-2Y.